A Fast Proof Checker for the Eunoia Logical Framework

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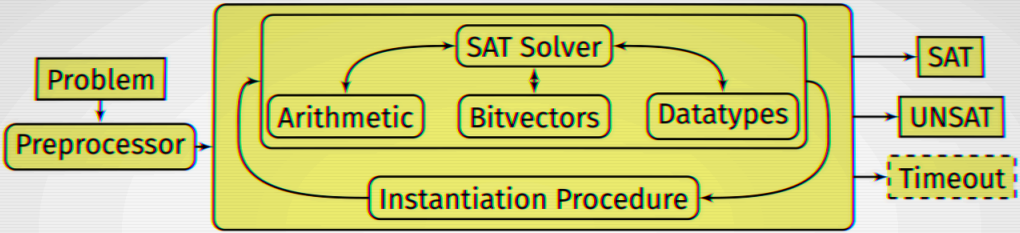
Proof Certificates For This?



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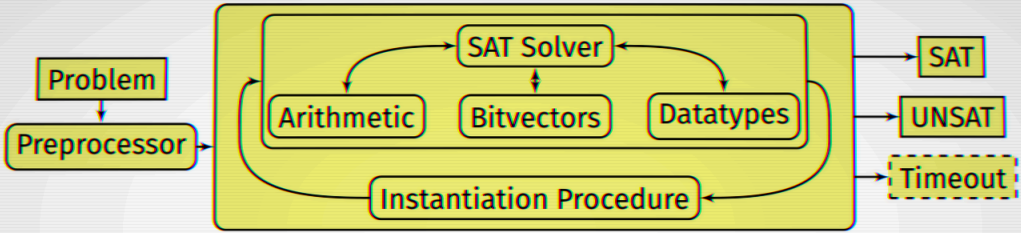

Two Ideas

LFSC A framework to model proof rules.
Problem: Hard to use.

Alethe A proof format based on the familiar SMT-LIB language.
Problem: English language specification.

Eunoia combines the good ideas of both!

Introducing Eunoia!

theory.eo

```
(declare-const Int Type)
(declare-consts <numeral> Int)

(declare-const not (-> Bool Bool))
(declare-const and (-> Bool Bool Bool) :right-assoc-nil true)

(declare-parameterized-const = ((A Type :implicit)) (-> A A Bool))

(declare-const BitVec (-> Int Type))
(declare-parameterized-const concat
  ((m Int :implicit) (n Int :implicit))
  (-> (BitVec n) (BitVec m) (BitVec (eo::add m n)))
)
```

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(include "theory.eo")

(declare-rule contra ((F Bool))
  :premises (F (not F))
  :conclusion false
)

(program pick ((F Bool) (Fs Bool :list) (n Int))
  :signature (Bool Int) Bool
  (((pick  F    0)    F)
   ((pick (and F Fs) n) (pick Fs (eo::add n -1))))
)
(declare-rule andE ((Fs Bool) (i Int))
  :premises (Fs)
  :args (i)
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Introducing Eunoia!

```
(reference "problem.smt2")

(include "rules.eo")

(declare-const F Bool)
(assume a1 (and F (not F)))

(step s1 (F)     :rule andE   :premises (a1) :args (0))
(step s2 (not F) :rule andE   :premises (a1) :args (1))
(step s3 (false) :rule contra :premises (s1 s2))
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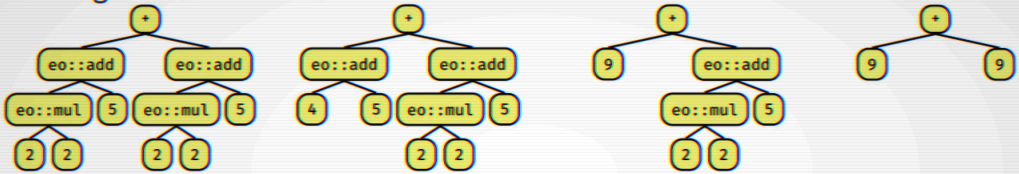
Now! Checking Eunoia Proofs Fast!

- **Ethos** is your bread and butter proof checker for Eunoia.
- Want to get your proofs checked fast? **Ethos!**
- Want to debug your Eunoia signatures? **Ethos!**
- Want to skip writing an Eunoia parser? **Ethos!**
- TCP from > 400k lines of cvc5 C++ to 12k + 10k lines of Eunoia and Ethos C++

Optimizations!

Caching of Evaluation Results



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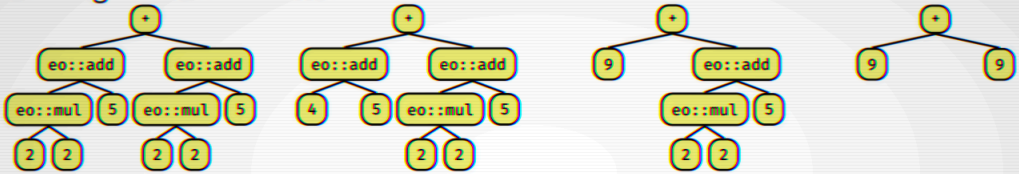
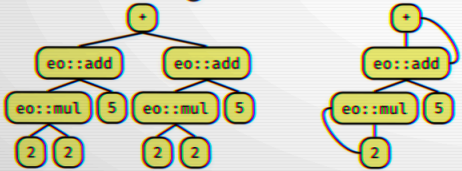


Perfect Sharing



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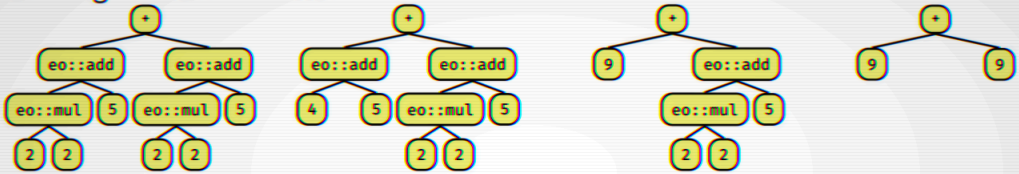

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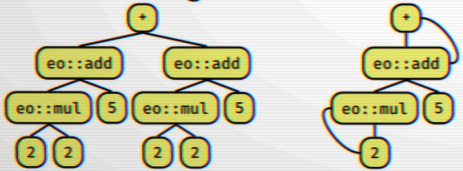

Keep Tails

`eo::list_erase_all … 4`

How Fast Is Ethos?

Evaluation

- Problem: do not measure solver behavior!

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- Problem: do not measure solver behavior!
- We evaluate on proofs generated by **cvc5**.
- Runtime cost: internal elaboration, calculus difference, checker speed.
- Formats: Alethe (with **Carcara**), **LFSC**, and CPC (with **Ethos**).

How Fast Is Ethos?

	Verified	Generation	Checking	Failures
CPC	20 813	85 868s	93 157s	571
Alethe	19 502	90 908s	13 869s	
Holes	867	7 172s	1 192s	1 211
LFSC	454	36s	42s	
Holes	19 275	81 459s	223 989s	127

UFLIRA
20 940 benchmarks, not known **sat**, solvable by cvc5 in 60s.
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	Verified	Generation	Checking	Failures
CPC	138 912	657 911s	219 267s	1 337
LFSC	705	36s	34s	
Holes	136 482	229 300s	221 911s	3 061

The LFSC numbers are distorted due to the large number of holes!

140 248 benchmarks, not known **sat**, solvable by cvc5 in 60s.
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Thank You!