Eunoia: A Framework for SMT Proof Calculi

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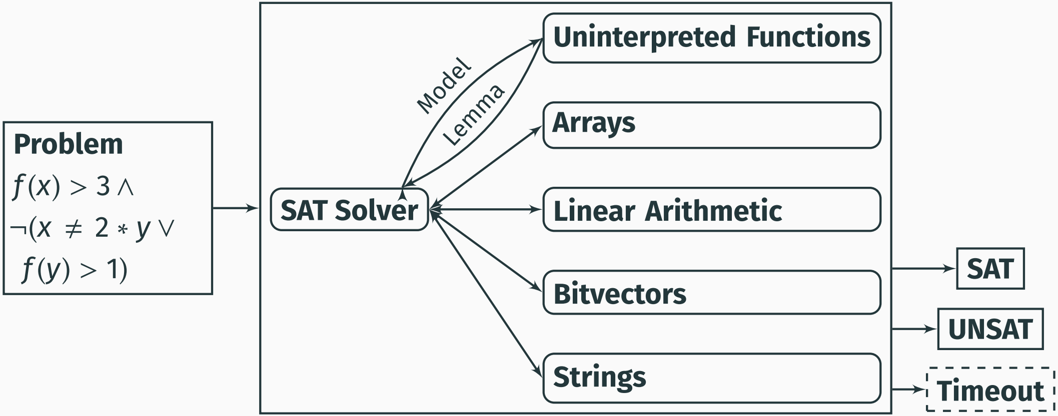
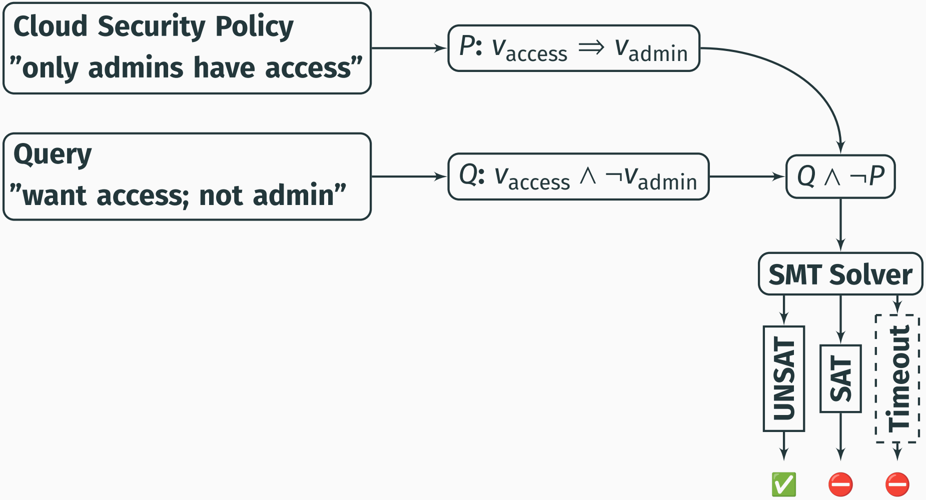
A Satisfiability Modulo Theories Solver



Application: AWS Zelkova



Application: AWS Zelkova

How can we trust the decision?

Testing

- SMT solvers are complex.
- Why are we doing formal methods at all?

Verify SMT Solver

- SMT solvers are complex.
- Might not reach acceptable performance.

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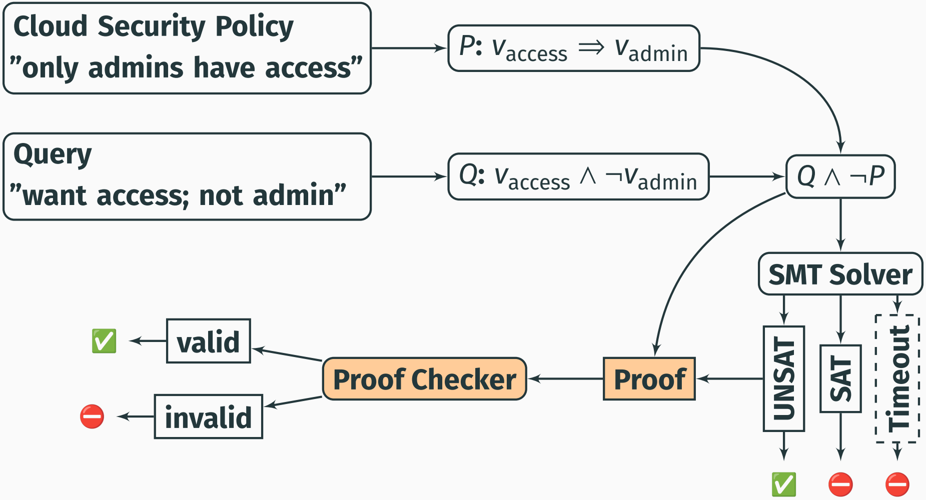
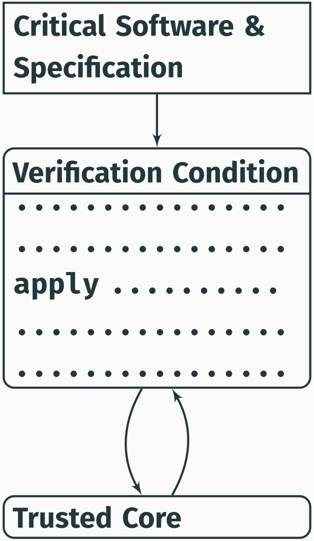
Proof Certificates

- Can be independently checked.
- Proof checkers are smaller.
- Removes the SMT solver from the critical path.
- Only needed for **UNSAT**.
- One checker per solver. 😥

Application: AWS Zelkova



Application: Verification With Isabelle/HOL



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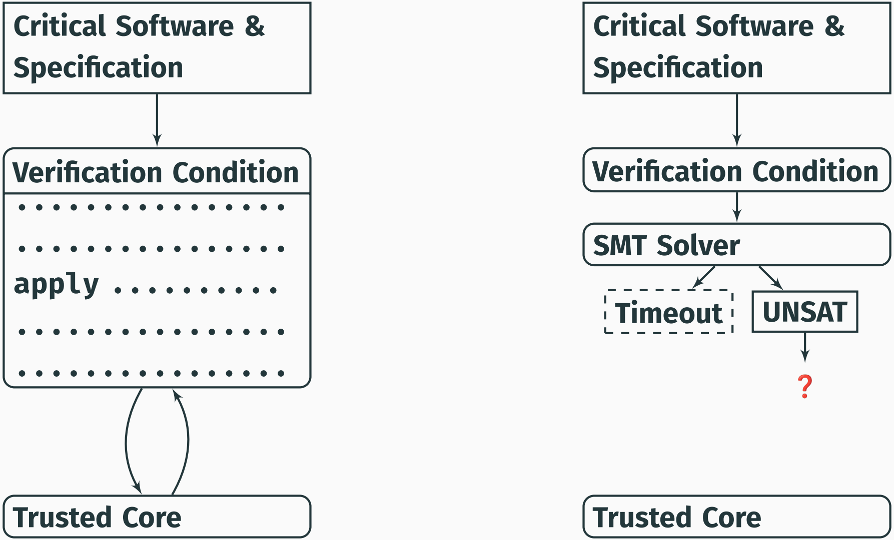

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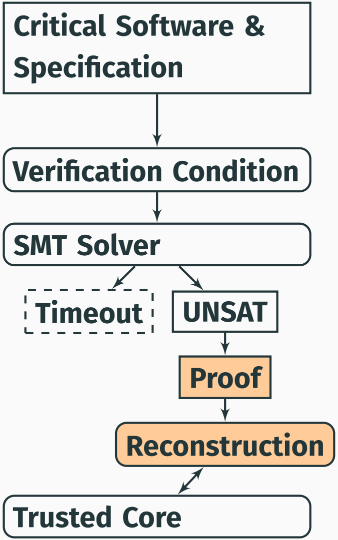
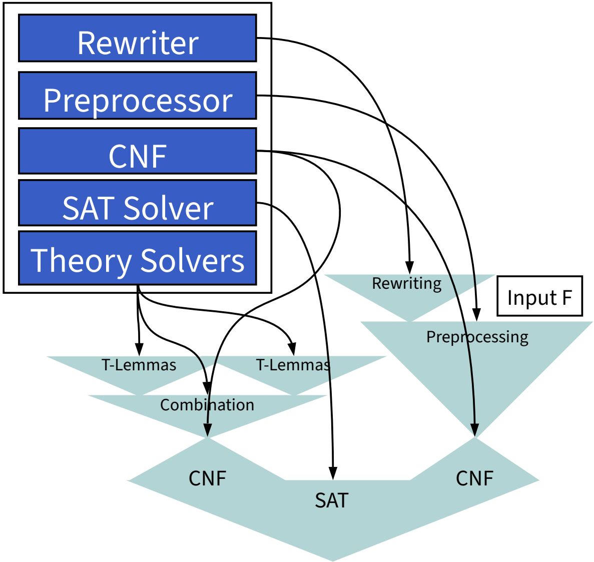


- Originally for Z3
- Then for veriT
  and cvc5
- Very labor intensive to build!
- Performance is critical.

What Goes Into a Proof



Two Proof Formats

Alethe
A general proof format.

+ Looks like SMT-LIB
+ Used! (cvc5, veriT, Isabelle, Carcara)
+ Well documented...
- ... in English
- Challenges with generality

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LFSC
A logical framework (with side conditions).

- \+ High performance checker
- \+ Declarative
- \- Hard to read
- \- Side conditions from another world
- \- Limited theories

Eunoia: Inspired by Alethe and LFSC

Goal 1
Look like SMT-LIB and Alethe.

By SMT people for SMT people!

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Goal 2
Provide a declarative language to specify proof rules for all SMT-LIB logics.
(that includes Bit Vectors).

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Allow fast checking of proofs against specified rules.

Non Goal
Rules can be specified freely. It is not necessary to prove them correct.
Not Curry-Howard correspondence based.

By SMT people for SMT people!

Eunoia: Example 1

```
(assume a1 (and true (= a b)))
(assume a2 (= b c))
(step s1 (= a b)          :rule andE  :premises (a1) :args (2))
(step s2 (= a c)          :rule trans :premises (s1 a2))
(step s3 (= (f a) (f c)) :rule cong  :premises (s2) :args (f))
```

Eunoia: Example 1 (Under The Hood)

$$\Gamma \vdash (\mathrm{cong}\ f\ (\mathrm{trans}\ (\mathrm{andE}\ 2\ (\mathrm{assume}\ (\top \wedge (a = b)))\ (\mathrm{assume}\ (a = b))))) : \mathrm{Proof}\ (f\ a = f\ b)$$

Eunoia: Example 1 (The Rules)

```
(declare-rule trans ((T Type) (a T) (b T) (c T))
   :premises ((= a b) (= b c))
   :conclusion (= a c)
)
(declare-rule cong ((T Type) (S Type) (a T) (b T) (f (-> S T)))
   :premises ((= a b))
   :args (f)
   :conclusion (= (f a) (f c))
)
```

Eunoia: Example 1 (The Rules)

```
(program select ((a Bool) (b Bool) (i Int))
    :signature (Int Bool) Bool
    (
        ((select 1 (and a b)) a)
        ((select 2 (and a b)) b)
    )
)
(declare-rule andE ((a Bool) (b Bool) (i Int))
   :premises ((and a b))
   :args (i)
   :conclusion (select i (and a b))
)
```

Eunoia: Example 1 (The Rules, Abstractly)

$$\Gamma \vdash \mathsf{trans} : \mathsf{Proof}\ a = b \to \mathsf{Proof}\ b = c \to \mathsf{Proof}\ a = c$$

$$\Gamma \vdash \mathsf{cong} : (f : T \to S) \to \mathsf{Proof}\ a = b \to \mathsf{Proof}\ (f\ a) = (f\ b)$$

$$\Gamma \vdash \mathsf{andE} : (i : \mathsf{Int}) \to \mathsf{Proof}\ a \land b \to \mathsf{Proof}\ (\mathsf{select}\ i\ (a \land b))$$

Eunoia: Example 2 (Recursion)

```
(program selectLast ((a Bool) (b Bool))
    :signature (Bool) Bool
    (
        ((selectLast (and a b)) (selectLast b))
        ((selectLast a)                      a )
    )
)
(declare-rule andLast ((a Bool))
   :premises (a)
   :conclusion (selectLast a)
)
```

Where do rules come from?

Cooperating Proof Calculus

- cvc5 specific
- for all standard theories
- 100% proof coverage in **safe** mode
- faster than old LFSC system

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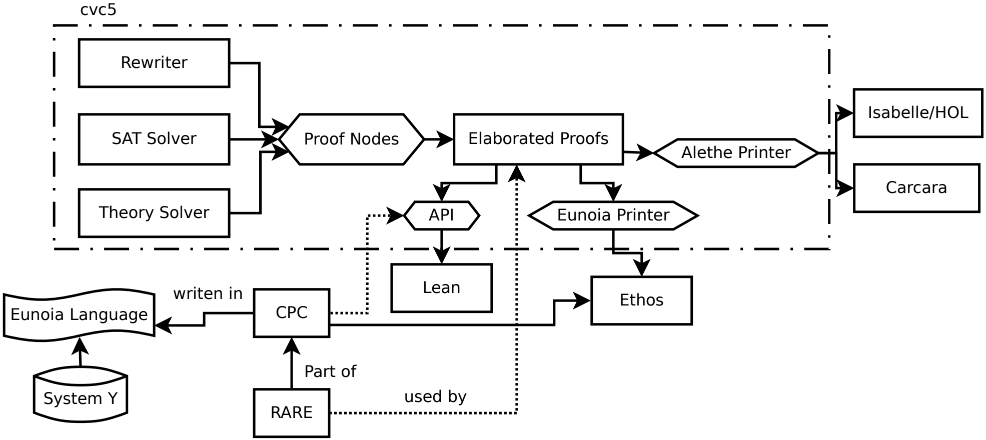
Alethe in Eunoia

- models the Alethe calculus
- with Eunoia syntax
- working proof of concept

The cvc5 Ecosystem



Checking Eunoia Proofs

Eunoia is

- a dependently typed programming language,
- that mixes data and computation freely,
- that allows divergent computations.
- Computations can throw exceptions
  - that can be cached!
  - i.e., we have observable effects.

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Oh, my...
how does that all work?

Let's look at Ethos!

Ethos: a Proof Checker, Not a Type Checker

Ethos checking model (roughly):

1. Check only that (constants, programs, rules) signature is well-formed.
2. Iterate over proof steps.
   - Observe that all terms have concrete type!

   2.1 Instantiate variables in types.

   2.2 Recurse into type constraints.

   2.3 Perform computations.

   - Divergence, exception: proof reject.

Upsides

- Correct!
- Fast.
- Easy to implement.
- Easy to extend.

Downsides

- Bugs in rules can be missed.
- Unexpected.
- Wasted work (e.g., function composition).

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J. Garrett Morris solving a problem.

μEunoia : System Ƴ + Core Eunoia

System Ƴ: *Decidable* Dependent Type Theory with explicit evaluation evidence.

μEunoia is not a subset of Eunoia.

μEunoia checking model (abstractly):

1. Write your signature in μEunoia (auto translation is future work).
2. Typecheck your signature.
3. Run modified Ethos on an **Eunoia** proof.
    - Divergence, exceptions: reject proof
    - Otherwise: output proof with evaluation evidence (μEunoia proof)
4. Typecheck your μEunoia proof.

Example

$$\text{zeros} : \Pi(n : \mathbb{Z}).\ \text{Vec}\,\text{Int}\,n$$

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$$\begin{aligned} \text{moreZeroes} &: (n : \mathbb{Z}) \to (m : \mathbb{Z}) \to & \text{let}\ p = \text{add}\ n\ m\ \langle 1 \rangle\ \text{in}\ \mathit{Vec}\ \mathbb{Z}\ p \\ \text{moreZeroes}\ n\ m &= & \text{dlet}\ p = \text{add}\ n\ m\ \langle 1 \rangle\ \text{in zeros}\ p \end{aligned}$$

Example

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&\text{moreZeroes}\ n\ m = && \text{dlet}\ p = \text{add}\ n\ m\ \langle 1\rangle\ \text{in zeros}\ p
\end{aligned}$$

$$\begin{aligned}
\text{theZeros} &: \text{Vec}\ \mathbb{Z}\ 12 \\
\text{theZeros} &= [\text{moreZeros}\ 9\ 3]\ \langle 4\rangle
\end{aligned}$$

The Project

- Goal: add enough to be “Eunoia”
- Deep Embedding in Agda!
- Substantial: $> 11\,000$ lines

What We Support

- Signatures
- Literals
- Overriding literal typing
- Non-linear matching
- Builtins
- Exceptions
- Special variable scoping
    - Declaration-wide scopes
    - “Quote” Arrow

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Status

100%	Language, Evaluation, Typing
100%	Unicity
100%	Decidability
100%	Progress
75%	Preservation
10%	Soundness Case Study

The State of Eunoia

Ethos Proof Checker
Experimentally deployed at AWS.

System description in progress (IJCAR).

CPC Rules
Stable and well tested.

Research paper in progress (CAV).

AletheInEunoia
Working prototype.

Alethe update paper in progress (IJCAR).

System Υ and μEunoia
Finishing touches needed.

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Downsides

- Does not give guarantees for the soundness of rules.
- i.e., it's trivial to define false : Proof $\bot$.
- Ethos is not verified.
- Does not simpify proof reconstruction.

The Future: Panproof

Elaborate **solver specific** rules into **common standard rules** on demand.

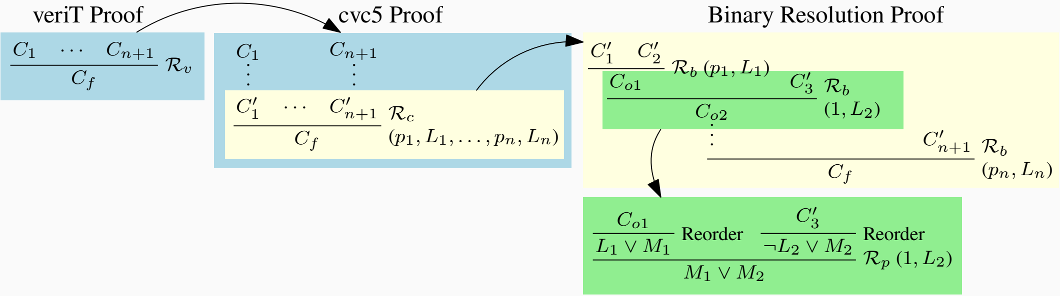
```
data ProofRule (@0 Γ : Signature) : Set where
  Rule :
    (name : String)
  → (infer     : (prems : List (Sequent Γ)) → Maybe (Sequent Γ)))
  → (elaborate : (prems : List (Sequent Γ)) → Maybe (Step Γ prems))
  → (@0 proof  : (prems : List (Sequent Γ))
        → (infer prems) == (checkStep (elaborate prems)))
  ---------------------------------------------------------------
  → ProofRule Γ
```

Prototype:

`https://github.com/hansjoergschurr/Panproof`


Thank You!

Panproof: Rule Example



Eunoia Variables: Declaration-Scoped

```
(declare-parameterized-const ubv_to_int
                ((m Int :implicit))
        (-> (BitVec m) Int))
(program fromBvAdd ((n Int) (bv (BitVec n)))
  :signature (Int (BitVec n)) Int
  ( ((fromInt n bv) (+ n (ubv_to_int bv)) )
)
```

"Quote" Arrow

$$\text{ex} : [n + m : \mathbb{Z}] \mapsto \text{BitVec } n$$

$$\text{ex } (1 + 2) : \text{BitVec } 1$$

- Every declaration has *n* variables.
  - `Vec Bool` n to mark assigned, free, bound variables
  - `Vec Term` n for typing, substitution.
- Kills De Bruijn indices 😞
- Matching with vectors that track free/bound variables 😎
- Spine-local type inference to assign types in applications.
- Big problem: `dlet` leaks variables to outer context!
  - Solution: Program calls must transfer variables into the caller context.

Rule Sketches

$$\frac{\Gamma \vdash M : A}{\Gamma \vdash_c M : A}$$

$$\frac{\Gamma \vdash A : \mathscr{T} \quad \Gamma, A \vdash_c M : B}{\Gamma \vdash \lambda\, M : \Pi\, A\, B}$$

$$\frac{\Gamma \vdash_c M : \mathscr{M}A \quad \Gamma \vdash M \twoheadrightarrow [n]\ \mathrm{return}\ V}{\Gamma \vdash \langle n \rangle : V \leftarrow M}$$

$$\frac{\Gamma \vdash A : \mathscr{T}}{\Gamma \vdash \mathscr{M}A : \mathscr{T}}$$

$$\frac{\Gamma \vdash A : \mathscr{T} \quad \Gamma, A \vdash_c B : \mathscr{T}}{\Gamma \vdash \Pi\, A\, B : \mathscr{T}}$$

$$\frac{\Gamma \vdash M : \mathrm{nlet}\ A\ B \quad \Gamma \vdash N : C \leftarrow A}{\Gamma \vdash [M]\ N : B\ [C]}$$

$$\frac{\Gamma \vdash M : A}{\mathrm{return}\ M : \mathscr{M}A}$$

$$\frac{\Gamma \vdash V : A \quad \Gamma \vdash_c M : \mathscr{M}A}{\Gamma \vdash V \leftarrow M : \mathscr{T}}$$

$$\frac{\Gamma \vdash_c M : \mathrm{nlet}\ A\ B \quad \Gamma \vdash N : C \leftarrow A}{\Gamma \vdash_c [M]\ N : B\ [C]}$$

$$\frac{\Gamma \vdash_c M : \mathscr{M}A \quad \Gamma \vdash_c B : \mathscr{T} \quad \Gamma, A \vdash_c N : \mathrm{wk}\ B}{\Gamma \vdash_c \mathrm{nlet}\ M\ N : B}$$

$$\frac{\Gamma \vdash_c M : \mathscr{M}A \quad \Gamma, A \vdash_c B : \mathscr{T} \quad \Gamma, A \vdash_c N : B}{\Gamma \vdash_c \mathrm{dlet}\ M\ N : \mathrm{nlet}\ M\ B}$$